BE - Commande Numérique

$$\rho(z)\frac{\partial^2 w}{\partial t^2}(z,t) = -\frac{\partial^2}{\partial z^2}\left[EI(z)\frac{\partial^2 w}{\partial z^2}(z,t)\right] - q(z,t)$$

$$x_1(z,t) = \frac{\partial^2 w}{\partial z^2}(z,t) \qquad e_1(z,t) = EI(z)\,x_1(z,t)$$

$$x_2(z,t) = \rho(z)\frac{\partial w}{\partial t}(z,t) \qquad e_2(z,t) = \frac{1}{\rho(z)}\,x_2(z,t)$$

$$\dot{x} = \begin{bmatrix}\dot{x}_1 \\ \dot{x}_2\end{bmatrix}$$

$$\underbrace{\rho(z)\frac{\partial^2 w}{\partial t^2}(z,t)}_{\dot{x}_2(z,t)} = -\frac{\partial^2}{\partial z^2}\left[EI(z)\cdot x_1(z,t)\right] - q(z,t)$$

$$\dot{x}_1 = \frac{\partial}{\partial t}\frac{\partial^2 w}{\partial z^2}(z,t) = \frac{\partial^2}{\partial z^2}\overbrace{\frac{\partial w}{\partial t}(z,t)}^{\frac{x_2(z,t)}{\rho(z)}} = \frac{\partial^2}{\partial z^2}\overbrace{\frac{x_2(z,t)}{\rho(z)}}^{e_2(z,t)}$$

$$x_2(z,t) = \rho(z)\frac{\partial w}{\partial t}(z,t) \Rightarrow \dot{x}_2(z,t) = \rho(z)\frac{\partial^2}{\partial t^2}w(z,t) = -\frac{\partial^2}{\partial z^2}\underbrace{EI(z)\,x_1(z,t)}_{e_1(z,t)} - q(z,t)$$

$$\dot{x} = \begin{bmatrix} 0 & \frac{\partial^2}{\partial z^2}1 \\ \frac{\partial^2}{\partial z^2}1 & 0\end{bmatrix}\left[e_1(z,t)\ \ e_2(z,t)\right] + \begin{bmatrix}0 \\ -q(z,t)\end{bmatrix}$$

$$\boxed{\dot{x} = \begin{bmatrix}\dot{x}_1 \\ \dot{x}_2\end{bmatrix} = \frac{\partial^2}{\partial z^2}\begin{bmatrix}0 & 1 \\ -1 & 0\end{bmatrix}\left[e_1(z,t)\ e_2(z,t)\right] + \begin{bmatrix}0 \\ -q(z,t)\end{bmatrix}}$$

2)

$$x_1(z,t) \simeq \phi(z)^T x_{1d}(t) \qquad \phi(z) = \begin{bmatrix} 2z^3 - 3z^2 + 1 \\ 3z^2 - 2z^3 \\ z^3 - 2z^2 + z \\ z^3 - z^2 \end{bmatrix}$$

$$x_2(z,t) \simeq \phi(z)^T x_{2d}(t)$$

$$e_1(z,t) = \phi(z)^T e_{1d}(t)$$

$$e_2(z,t) = \phi(z)^T e_{2d}(t)$$

$$\dot{x}_1(z,t) = \frac{\partial^2}{\partial z^2}e_2(z,t) \Leftrightarrow \overbrace{\phi(z)^T\dot{x}_{1d}(t)}^{\sim x_1(z,t)} = \frac{\partial^2}{\partial z^2}\overbrace{\phi^T(z)e_{2d}(t)}^{\sim e_2(z,t)}$$

$$\phi(z)\cdot\phi^T(z)\,\dot{x}_{1d}(t) = \phi(z)\frac{\partial^2}{\partial z^2}\phi^T(z)\,e_{2d}(t)$$

$\int \cdot\, dz$

$$\int_0^L \phi(z)\phi^T(z)\,\dot{x}_{1d}(t)\,dz = \int_0^L \phi(z)\frac{\partial^2}{\partial z^2}\phi^T(z)\,e_{2d}(t)\,dz$$

à donner à Matlab, il fait pas seul

$$\Leftrightarrow \underbrace{\int_0^L \phi(z)\phi^T(z)\,dz}_{E}\ \dot{x}_{1d}(t) = \underbrace{\int_0^L \phi(z)\frac{\partial^2}{\partial z^2}\phi^T(z)\,dz}_{D}\ e_{2d}(t)$$

$$\dot{x}_2(z,t) = -\frac{\partial^2}{\partial z^2} e_1(z,t) - q(z,t)$$

$$\phi^T(z)\,\dot{x}_{2d}(t) = -\frac{\partial^2}{\partial z^2}\phi^T(z) e_{1d}(t) - q(z,t)$$

$\cdot\phi(z)$

$$\phi(z)\phi^T(z)\,\dot{x}_{2d}(t) = -\phi(z)\frac{\partial^2}{\partial z^2}\phi^T(z) e_{1d}(t) - \phi(z) q(z,t)$$

$\int_0^L \cdot\, dz$

$$\int_0^L \phi(z)\phi^T(z)\dot{x}_{2d}(t)\,dz = \int_0^L -\phi(z)\frac{\partial^2}{\partial z^2}\phi^T(z) e_{1d}(t)\,dz - \overbrace{\int_0^L \phi(z) q(z,t)\,dz}^{F_{ext}}$$

$$\Leftrightarrow \underbrace{\int_0^L \phi(z)\phi^T(z)\,dz}_{E}\,\dot{x}_{2d}(t) = \underbrace{-\int_0^L \phi(z)\frac{\partial^2}{\partial z^2}\phi^T(z)\,dz\, e_{1d}(t)} - \underbrace{\int_0^L \phi(z) q(z,t)\,dz}_{F_{ext}}$$

$$-\int_0^L \phi(z)\frac{\partial^2}{\partial z^2}\phi^T(z)\,dz\, e_{1d}(t) = -\int_0^L \phi(z)\frac{\partial}{\partial z}\overbrace{\frac{\partial}{\partial z}\phi^T(z) e_{1d}(t)}^{e_1(z,t)\,\simeq\,\phi^T(z)e_{1d}(t)}\,dz$$

$$= -\left(\left[\phi(z,t)\frac{\partial\phi^T}{\partial z}(z,t)\right]_0^L - \int_0^L \frac{\partial\phi(z,t)}{\partial z}\frac{\partial}{\partial z}\phi^T(z,t)\,dz\right)\underbrace{e_{1d}(t)}_{e_1(z,t)}$$

$$= -\left(\phi(L,t)\frac{\partial\phi^T}{\partial z}(L,t) - \underbrace{\phi(0,t)\frac{\partial\phi^T}{\partial z}(0,t)}_{=0} - \int_0^L \frac{\partial\phi}{\partial z}(z,t)\frac{\partial\phi^T}{\partial z}(z,t)\,dz\right) e_{1d}(t)$$

$$= -\left(\phi(L,t)\underbrace{\frac{\partial\phi^T}{\partial z}(L,t)\, e_{1d}(t)}_{u(t)} - \int_0^L \frac{\partial}{\partial z}\phi(z,t)\frac{\partial\phi^T}{\partial z}(z,t)\,dz\, e_{1d}(t)\right) \Rightarrow D$$

$$= -\phi(L,t)u(t) + \left[\frac{\partial}{\partial z}\phi(z,t)\phi^T(z,t)\right]_0^L - \int_0^L \frac{\partial^2}{\partial z^2}\phi(z,t)\phi^T(z,t)\,dz$$

$$= -\phi(L,t)u(t) + \left[\frac{\partial}{\partial z}\phi(L,t)\phi(L,t) - \int_0^L \frac{\partial^2}{\partial z^2}\phi(z,t)\phi^T(z,t)\,dz\right] e_{1d}(t)$$

$$= \underbrace{\left[-\phi(L,t)\frac{\partial\phi^T}{\partial z}(L,t) + \frac{\partial}{\partial z}\phi(L,t)\phi(L,t) - \int_0^L \frac{\partial^2}{\partial z^2}\phi(z,t)\phi^T(z,t)\,dz\right]}_{-D} e_{1d}(t)$$

$$-D + \phi(L,t)\frac{\partial\phi^T}{\partial z}(L,t) = \phi(L,t)\frac{\partial\phi^T}{\partial z}(L,t) - \int_0^L \frac{\partial^2}{\partial z^2}\phi(z,t)\phi^T(z,t)\,dz \quad \text{Jsp}$$

$$y(t) = -e_2(L,t)$$

$$\downarrow$$

$$y(t) = -\phi^T(L,t)\, e_{2d}(t)$$

$$e_1(\zeta,t) = \overbrace{EI(\zeta)}^{=1}\, x_1(\zeta,t) \approx \phi^T(\zeta)\, e_d(t) = \phi^T(\zeta)\, x_{1d}(t)$$

$$e_2(\zeta,t) = \underbrace{\frac{1}{\rho(\zeta)}}_{=1}\, x_2(\zeta,t) \quad \text{etc}$$

3)
$$\dot{x}_d(t) = \begin{bmatrix} \dot{x}_{1d}(t) \\ \dot{x}_{2d}(t) \end{bmatrix} = \begin{bmatrix} E^{-1} D\, e_{2d}(t) \\ -E^{-1} D^T e_{1d}(t) - E^{-1}\phi(L)u(t) - E^{-1}F_{ext} \end{bmatrix}$$

$$= \begin{bmatrix} E^{-1} D\, x_{d2}(t) \\ -E^{-1} D\, x_{d1}(t) - E^{-1}\phi(L)u(t) - \overbrace{E^{-1}F_{ext}}^{=0} \end{bmatrix}$$

$$= \underbrace{\begin{bmatrix} 0 & E^{-1}D \\ -E^{-1}D & 0 \end{bmatrix}}_{A} \begin{bmatrix} x_{1d}(t) & x_{2d}(t) \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ -E^{-1}\phi(L) \end{bmatrix}}_{B} u(t)$$

$$y(t) = \underbrace{\begin{bmatrix} 0 \\ -\phi^T(L,t) \end{bmatrix}}_{C} \begin{bmatrix} x_{1d}(t) & x_{2d}(t) \end{bmatrix}$$

$$\iint_0^t c^3 - 2c^2 + c \, dc^2 = \int_0^\zeta \left[\frac{1}{4}c^4 - \frac{2}{3}c^3 + \frac{1}{2}c^2\right]_0^t d\zeta$$

$$= \int_0^\zeta \frac{1}{4}t^4 - \frac{2}{3}t^3 + \frac{1}{2}t^2 \, d\zeta = \left[\frac{1}{5\cdot 4}t^5 - \frac{2}{3\cdot 4}t^4 + \frac{1}{6}t^3\right]_0^\zeta = \frac{1}{20}\zeta^5 - \frac{1}{6}\zeta^4 + \frac{1}{6}\zeta^3$$

$$\frac{\zeta^3(3\zeta^2 - 10\zeta + 10)}{60}$$

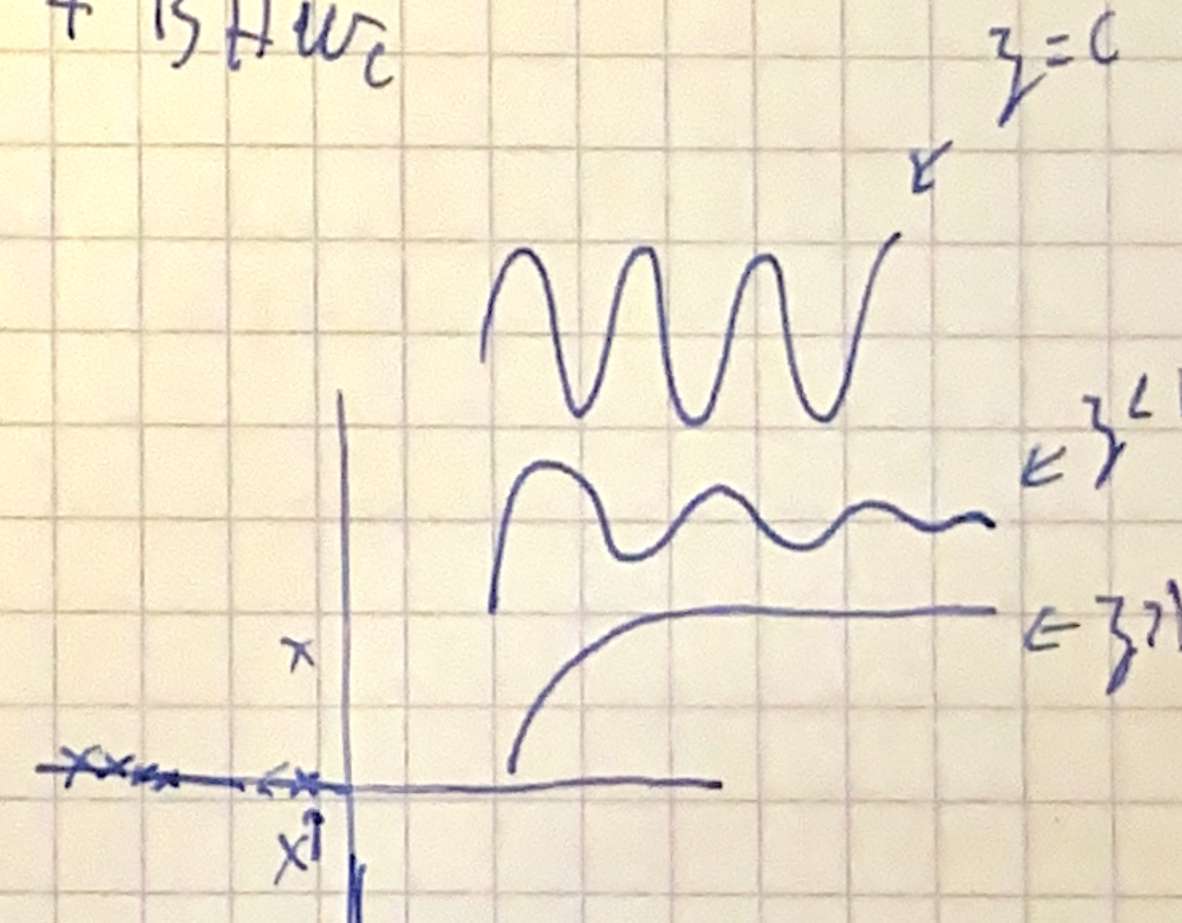
$$\dot{x} = Ax + Bu \qquad u = -ky + Hw_c$$
$$y = Cx$$

$$\dot{x} = Ax + B(-ky + Hw_c),$$

$$= Ax + B(-kCx + Hw_c) = (A - BkC)x + BHw_c$$

$$\dot{x} = \tilde{A}x + \tilde{B}w_c, \qquad \tilde{A} = A - Bk\,C \;\; \text{scalaire}$$
$$y = Cx \qquad \tilde{B} = BH$$

$\uparrow$ scalaire



$k = 2$;

$$\frac{1}{\tau} = w \qquad 2\pi f = w$$

$$f_e > 2 f_{max} \Rightarrow \qquad f_{max} = \frac{1}{2\pi\tau}$$

d(1)

$u(t) = -Kx(t) + Hw_i(L,t)$ $\qquad x(t) = \phi^T(\xi)\, X_d(t)$

$\dot{X}_d(t) = A_d x_d(t) + B_d u(t)$

$y(t) = C_d x_d(t)$

$\dot{X}(t) = Ax(t) + Bu(t)$ $\Rightarrow$ $\dot{x}(t) = Ax(t) + B[-Kx(t) + Hw_i(L,t)]$

$y(t) = Cx(t)$ $\qquad = \underbrace{(A - BK)}_{\tilde{A}} x(t) + \underbrace{BH}_{\tilde{B}} w_i(L,t)$

$\dot{x}(t) = 0 \Rightarrow (A - BK)x^*(t) = -BHw_i(L,t)$ $\qquad w_i = Lx^*$

$\Leftrightarrow -(A-BK)^{-1} BHw_i(L,t) = x^*$

$\Leftrightarrow \underbrace{Lx^*}_{w_i} = -C[A-BK]^{-1} BHw_i$ $\qquad x^* = w_i$

$\Leftrightarrow I = -C[A-BK]^{-1}BH$

$H = [-C(A-BK)^{-1}B]^{-1}$

$F_{ext} = \phi(L)\cdot q(t) = \begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ -1 \end{bmatrix}^T$